Series 2

Exercise 1

Use image theory and the fact that the tangentiel E fields in zero on a PEC to:

1. Find the E field radiated by a Hertzian dipole placed at a height *h* above a ground plane, supporting a current *I* and oriented orthogonally to the ground ***Adivice***: place the ground plane at *z = 0* and the dipole at point *P (0, 0, h)* and work in spherical cooridnates *(r, θ)*.

2. Sketch the *E* field as a function of θ.

Exercise 2

The electric field transmitted by an antenna is given by:

$$\vec{\mathbf{E}} = \hat{\mathbf{e}}_{\boldsymbol{\theta}} C \cos^n \theta \sin\theta \exp(-jkr)/r$$

where *C* is a constant, n is an integer and the expression is valid in the far field.

Dans cette région, le champ magnétique peut toujours être calculé comme:

$\boldsymbol{H} = (1/Z_0)\hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{E}$ où Z_0 is a constant, the characteristic impedance of the medium

1. Find the expression of the raidated power density, given by teh amplitude of the Poynting vector $p(r,\theta,\varphi) = \left|\vec{\mathbf{S}}\right| = \left|\vec{\mathbf{E}} \times \vec{\mathbf{H}}^*\right|$
2. Find the total radiated power $P_{rad}(r = R)$, integrating the power density $p(r = R,\theta,\varphi)$ on the spherical surface of radius R Does this power depend on R ?

NOTE: if you find the math difficult, try at last the case $n = 0$

3. Directivity. Compute the directivity, by first computing the mean of the radiated power density, $p_{iso}(r = R)$ existing on a surface of radius R, as the ratio between the total radiated power and the surface of the sphere. The directivity is then given by $D(\theta,\varphi) = p(r = R,\theta,\varphi)/p_{iso}(r = R)$. Find its maximum value, $D_{\max}$ as a function of *n*. Give the results in dB